nite; in the second case, the winter shadow is infinite and the summer shadow is finite.

In formula (A), δ and n can have any values within proper limits.

Also solved by *W. W. LANDIS*, and *J. SCHEFFER*.

59. Proposed by J. A. CALDERHEAD, M. Sc., Professor of Mathematics, Curry University, Pittsburg, Pa.

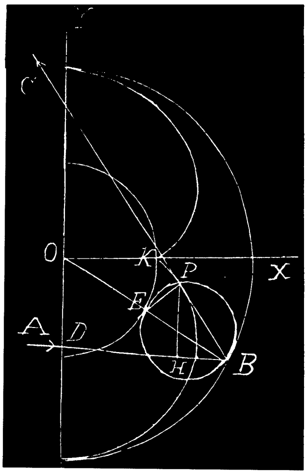
When a cylindrical china jar, standing upon the ground, receives the sun's rays obliquely, a bright curve is observed to form itself at the bottom of the jar, and it is found that the shape and dimensions of this curve are not affected by the varying elevations of the sun: account for this latter circumstance, and determine the nature of the bright curve. [From *Parkinson's Optics*.]

Solution by ALFRED HUME, C. E., D. Sc., Professor of Mathematics, University of Mississippi, University, Mississsppi.

All rays striking any element of the cylindrical surface lie in a vertical plane. Their reflections form the other face of the dihedral angle whose bisector passes through the axis of the cylinder. These reflected rays intersect the base of the cylinder in a straight line. There is thus formed a system of lines, and the bright curve observed is their envelope. The altitude of the sun does not affect the position of the vertical planes; and, therefore, the intersections with the bottom of the jar are unchanged, and the continual intersection of the consecutive lines so formed produces a curve invariable as to its shape and size.

The bright curve is the caustic by reflection for the circle, the incident rays being parallel. The following general property of caustics by reflection for parallel rays is established in *Price's Infinitesimal Calculus*: "The distance from the incident point in the reflecting curve to the point of intersection of two consecutive reflected rays, is equal to one-fourth of the chord of the circle of curvature at the point of incidence which is parallel to the incident ray."

A. Take the center of the circle as the origin, the X-axis parallel to the incident rays, the Y-axis perpendicular to them. Let AB be an incident ray, BC its reflection, the angle between them being 2θ. Take BP along BC equal to one-half of DB, D being the intersection of AB with the Y-axis. Then, according to the principle quoted above, P is a point of the caustic. To find the locus of P: Draw OB, denoting it by a. From P drop a perpendicular to AB meeting it at H. Denoting the coördinates of P by x and y,



$$x = DB - HB = a\cos\theta - \tfrac{1}{2}a\cos\theta\cos2\theta,$$

$$y = OD - PH = a\sin\theta - \tfrac{1}{2}a\cos\theta\sin2\theta.$$

From these $x = \tfrac{3}{4}a\cos\theta - \tfrac{1}{4}a\cos3\theta$,

and $y = \tfrac{3}{4}a\sin\theta - \tfrac{1}{4}a\sin3\theta$.

These may be written

$$x=(\tfrac{1}{2}a+\tfrac{1}{4}a)\cos\theta-(\tfrac{1}{4}a)\cos[(\tfrac{1}{2}a+\tfrac{1}{4}a)/(\tfrac{1}{4}a)]\theta,$$

$$\text{and } y=(\tfrac{1}{2}a+\tfrac{1}{4}a)\sin\theta-(\tfrac{1}{4}a)\sin[(\tfrac{1}{2}a+\tfrac{1}{4}a)/(\tfrac{1}{4}a)]\theta.$$

These are the well-known equations of an epicycloid, the radii of the fixed and rolling circles being $\frac{1}{2}a$ and $\frac{1}{4}a$ respectively.

B. The following geometrical solution is very much like one given in *Wood's Optics*, and was suggested by it.

Referring to the same figure, erect at P a perpendicular to CB, meeting OB at E. Comparing the similar triangles EPB and ODB, $ED : DB=BP : BD =1 : 2$.

If, then, upon EB, the half of OB, as diameter, a circumference be drawn its intersection with CB will be a point of the caustic. With O as center and EO as radius describe a semi-circle, intersecting the X-axis at K.

The $\angle EOK=\theta$, and arc $EK=\frac{1}{2}a\theta$.

Also, since $\angle EBP=\theta$, the angle at the center measured by arc $EP=2\theta$; and arc $EP=\frac{1}{4}a.2\theta=\frac{1}{2}a.\theta$.

Hence arc EP=arc EK.

The locus of P is, therefore, generated by the circle EPB rolling on the circle EK, the points P and K being originally in contact.

Of course the problem may be solved without assuming the property quoted from *Price*. In *Rice and Johnson's Differential Calculus* an excellent solution is outlined.

Also solved by *C. W. M. BLACK*, *S. H. WRIGHT*, and *B. F. FINKEL*.

PROBLEMS FOR SOLUTION.

ARITHMETIC.

97. **Proposed by J. A. CALDERHEAD, M. Sc., Professor of Mathematics, Curry University, Pittsburg, Pa.**

In what time will $4000 amount to $5134.96, interest at 6% payable annually?

*** Solutions of these problems should be sent to B. F. Finkel, not later than July 10.

GEOMETRY.

97. **Proposed by CHAS. C. CROSS, Libertytown, Md.**

Prove by pure geometry: The radius of a circle drawn through the centers of the inscribed and any two escribed circles of a triangle is double the radius of the circumscribed circle of the triangle.

98. **Proposed by EDW. R. ROBBINS, Master in Mathematics and Physics, Lawrenceville School, Lawrenceville, N. J.**

Construct a circle which shall pass through two given points and touch a given circle, (1) when the distance between the points is less than the diameter of the circle, and (2) when it is greater.

*** Solutions of these problems should be sent to B. F. Finkel, not later than July 10.